

## Ratios and Proportions

A ratio is a relationship between two numbers of the same kind (e.g., Objects, persons, units of some identical dimension)

Ratio expressions:

A 2 to 5 ratio can be expressed as 2:5 (with a colon) or  $\frac{2}{5}$  (a fraction).

A proportion is an equation with two ratios set equal to each other.

Example:

$$\frac{a}{b} = \frac{c}{d} \Rightarrow bd\left(\frac{a}{b}\right) = \left(\frac{c}{d}\right)bd \Rightarrow ad = bc$$

Which is the cross product

$$\frac{a}{b} = \frac{c}{d}$$

Example:  $\frac{2}{3} = \frac{4}{6}$

$$\begin{aligned} 6(2) &= 4(3) \\ 12 &= 12 \end{aligned}$$

$$\frac{2}{3} \neq \frac{1}{2} \text{ why}$$

$$\begin{aligned} 2(2) &\neq 3(1) \\ 4 &\neq 3 \end{aligned}$$

## Solving a Proportion

$$\frac{4}{5} = \frac{x}{10}$$

$$4(10) = 5x$$

$$\frac{40}{5} = \frac{5x}{5}$$

$$8 = x$$

$$\frac{x+2}{4} = \frac{2}{3}$$

$$3(x+2) = 2(4)$$

$$3x+6 = 8$$

$$-6 \quad -6$$

$$3x = 2$$

$$x = \frac{2}{3}$$

$$\frac{1}{2x} = \frac{5}{9}$$

$$9(1) = 5(2x)$$

$$9 = 10x$$

$$\frac{9}{10} = x$$

A chocolate chip cookie recipe calls for 1 cup of brown sugar and it yields 36 cookies. If you want to make 24 cookies instead, how much brown sugar do you need?

Let the ratio be cups to yield in the recipe.

$$\frac{1}{36} = \frac{x}{24} \Rightarrow 24 = 36x \Rightarrow \frac{24}{36} = \frac{x}{1} \Rightarrow x = \frac{2}{3}$$

So you should use  $\frac{2}{3}$  cup of brown sugar in your new recipe

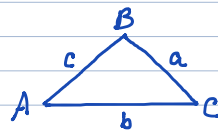
## Similar Triangles

Same shape, different sizes

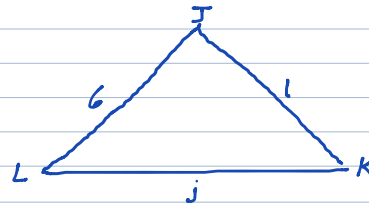
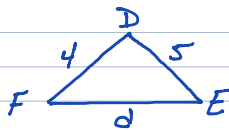
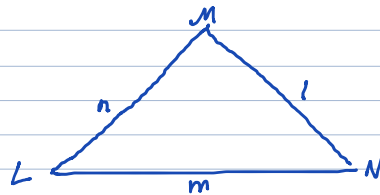
Corresponding angles are equal

Length of corresponding sides are proportional

Triangles ABC and LMN are similar triangles



$$\frac{a}{l} = \frac{b}{m} = \frac{c}{n}$$



$$\frac{4}{6} = \frac{5}{l}$$

$$4l = 30 \quad \text{Cross multiply}$$

$$l = \frac{30}{4} \quad \text{divide by 4}$$

$$= \frac{15}{2} = 7.5 \quad \text{Simplify}$$

Write a proportion with ratios of corresponding sides

Cross multiply

divide by 4

Simplify

Jane can complete a job in 4 hours working alone and Jill can complete the job in 3 hours working alone. How long would it take to complete the job working together?

We must find the portions that they each can do in one hour and set the sum equal to the portion they can do together in one hour. Let  $x$  be the time working together to complete the job.

$$\frac{1}{4} + \frac{1}{3} = \frac{1}{x}$$

$$12x \left( \frac{1}{4} + \frac{1}{3} \right) = 12x \left( \frac{1}{x} \right)$$

$$\frac{12x}{4} + \frac{12x}{3} = \frac{12x}{x}$$

$$3x + 4x = 12$$

$$7x = 12$$

$$x = \frac{12}{7} \approx 1.7 \text{ hrs.}$$